IFLST 0 Induction

Prove by Induction:

0.1 $1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + .. + n \cdot 2^n = 2 + (n-1)2^{n+1}$

0.2 $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$,

0.3 $1^2 + 2^2 + 3^2 + \ldots n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$,

0.4 $1^3 + 2^3 + 3^3 + \ldots n^3 = \frac{1}{4}(1+n)^2 n^2$,

0.5 $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n \cdot (n+1) = \frac{1}{3}n(n+1)(n+2)$,

0.6 $2^3 + 4^3 + \ldots (2n)^3 = 2n^2(n+1)^2$,

0.7 $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots n \cdot n! = (n+1)! - 1$,

0.8 $5n \leqslant n^2 - 3$ for $n \geqslant 6$,

0.9 $n^3 < 4^n$,

0.10 $3^n > n2^n$,

0.11 Bernouli inequality $(1+a)^n \geqslant 1 + n \cdot a, \quad a > -1, \quad n \in \mathbb{N}$,

0.12 $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + .. + \frac{1}{\sqrt{n}} > \sqrt{n}, \quad n \geqslant 2$,

0.13 $8 | 5^n + 2 \cdot 3^{n-1} + 1$,

0.14 $11 | 2^{6n+1} + 3^{2n+2}$,

0.15 $133 | 11^{n+2} + 12^{2n+1}$

0.16 $9 | 4^n + 24n - 1$.

0.17 The sequence a_n is given by $a_0 = 1$ $a_n = \frac{a_{n-1}}{2a_{n-1}+1}$. Find and prove a formula for a_n.

0.18 Prove there is infinite number of prime numbers.

0.19 Consider n points, for every two of them there is an arrow from one to another. A center is a point from which we can get to any other point in two steps going in the direction on arrows. Prove that a center always exists.

0.20 We will prove inductively that all cats are in the same color. Take a group consisting one cat. All cats in this group are in the same color. Induction step: assume any group of k cats are in the same color. Consider group of $k+1$ cats. Take one of them, by induction assumption the are in the same color. Put it back an take another one, the rest is again of the same color. So all $k+1$ cats are in the same color. The prove is complete, what is wrong?